

# Electric monopoles in generalised $B \wedge F$ theories

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## Abstract

A tensor product generalisation of  $B \wedge F$  theories is proposed with a Bogomol'nyi structure. Non-singular, stable, finite-energy particle-like solutions to the Bogomol'nyi equations are studied. Unlike Yang-Mills(-Higgs) theory, the Bogomol'nyi structure does not appear as a perfect square in the Lagrangian. Consequently, the Bogomol'nyi energy can be obtained in more than one way. The added flexibility permits electric monopole solutions.

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## 1 Introduction.

It is well-known that a Bogomol'nyi structure in a Lagrangian field theory frequently yields classical, non-singular, stable, finite-energy solutions to the variational field equations. Moreover, these particle-like solutions—called Bogomol'nyi solitons—appear to have far fewer quantum corrections than might

usually be expected. For example, the classical mass spectrum for Bogomol'nyi solitons has no quantum correction; this is due to a general relationship between supersymmetry and the Bogomol'nyi structure [1][2]. Also, for some Bogomol'nyi solitons (e.g., the BPS magnetic monopole) the quantum corrections to the scattering differential cross-section have been found to be remarkably and unexpectedly small [3].

In this letter we study a generalisation of the  $B \wedge F$  topological field theories introduced by Horowitz [4]. These theories are examples of generally covariant topological gauge field theories. The generalisation utilizes a tensor-product structure in the Lagrangian to produce a Bogomol'nyi structure. Solitons analogous to the BPS magnetic monopole in Yang-Mills-Higgs theory are found. Unlike Yang-Mills-Higgs theory, however, our Lagrangian does not rely on a metric structure. The metric structure used to define the Hodge star-operator in Yang-Mills-Higgs theory is responsible for turning the Bogomol'nyi soliton into a magnetic monopole. Without the metric in the tensor-product theory we are able to construct explicit electric monopole solutions to the field equations.

## 2 TFTs and Bogomol'nyi structures

The Lagrangian field theory that forms the basis of our work is given by

$$\begin{aligned} \mathcal{L}(A, B) = \int_{M_4} & \langle (H^A \otimes I_E) \wedge (I_E \otimes K^B) \rangle - \frac{1}{2} \langle (I_E \otimes K^B)^2 \rangle \\ & + (A \leftrightarrow B, H^A \leftrightarrow K^B) \end{aligned} \quad (1)$$

where  $H^A$  and  $K^B$  are gauge field curvatures over a four-manifold,  $M_4$ , taking values in the adjoint bundle  $E$  over  $M_4$ .  $I_E$  is the identity transformation on the adjoint bundle,  $E \rightarrow M_4$ . The form of this Lagrangian is based on the topological gauge field theories studied some time ago by Horowitz [4] and the theories of Baulieu and Singer [5]. The four-dimensional tensor product Lagrangian gauge field theory contains source-free electrodynamics and Yang-Mills theory [6]. Our interest in this letter lies in topological solitons with rest

mass. Therefore without loss of generality, we can restrict our investigation to stationary topological solitons. This leads us to dimensionally-reduce the four-dimensional theory using a gauge-symmetry in time [7]. We denote the four-manifold quotiented by the time-symmetry by  $M_3$ . Let the gauge group equal  $U(n)$ .  $P$  is a principal  $U(n)$ -bundle over the three-manifold  $M_3$ . Denote the space of connections on  $P$  by  $\mathcal{A}(P)$ . We represent by  $E$  the vector bundle over  $M_3$  associated to  $P$  by the adjoint representation. For each connection  $A \in \mathcal{A}(P)$  there is an exterior covariant derivative  $D^A$  acting on sections of  $E$ . The covariant derivative defines a curvature  $H^A$  for the vector bundle  $E$  by  $D^A D^A s = H^A s$ , where  $s$  is a section of the vector bundle  $\pi : E \rightarrow M_3$ . The curvature  $H^A$  can be interpreted as a 2-form on  $M_3$  taking values in  $E$ . We also introduce an equivariant Lie algebra valued Higgs field,  $\Phi_A$ , on  $M_3$  paired with the connection  $A$ . In dimensional reduction the Higgs field arises as the component of the vector potential in the direction of the gauge symmetry [7]. Our starting point therefore is the energy functional dimensionally-reduced from the Lagrangian (1). The energy functional  $2\pi\mathcal{E}(A, B, \Phi_A, \Phi_B)$  is given by

$$\begin{aligned} \int_{M_3} < (I_E \otimes K^B) \wedge (I_E \otimes D^B \Phi_B) > - \int_{M_3} < (I_E \otimes K^B) \wedge (D^A \Phi_A \otimes I_E) > \\ - \int_{M_3} < (H^A \otimes I_E) \wedge (I_E \otimes D^B \Phi_B) > + (A \leftrightarrow B, H^A \leftrightarrow K^B, \Phi_A \leftrightarrow \Phi_B). \end{aligned} \quad (2)$$

In the expression (2) there are two curvatures  $H^A, K^B$  and two Higgs fields  $\Phi_A, \Phi_B$  corresponding to two connections  $A, B \in \mathcal{A}(P)$ . We assume that there is an invariant positive-definite inner product  $< >$  on  $E_x$  which varies continuously with  $x \in M_3$ . The last term in (2) symmetrises the energy functional in the dependent fields. The energy functional is complicated, but as we shall see inherits useful geometric structure from the four-dimensional theory presented in [6][8]. In coordinate notation the energy functional  $2\pi\mathcal{E}(A, B, \Phi_A, \Phi_B)$  can be rewritten as

$$\begin{aligned} \int_{M_3} K_{[ij]}^a (D_{[k]}^B \Phi_B)^b \text{tr}(T^a T^b) - \int_{M_3} K_{[ij]}^a (D_{[k]}^A \Phi_A)^b \text{tr}(T^a I_E) \text{tr}(T^b I_E) \\ - \int_{M_3} H_{[ij]}^a (D_{[k]}^B \Phi_B)^b \text{tr}(T^a I_E) \text{tr}(T^b I_E) + (A \leftrightarrow B, \Phi_A \leftrightarrow \Phi_B). \end{aligned} \quad (3)$$

Square brackets denote skew-symmetrization, and Latin subscripts run from 1 to 3. Since the gauge group is  $U(n)$  we have used the Killing-Cartan form for the bundle inner product,  $\langle \rangle$ , normalised so that  $\langle I_E^2 \rangle = 1$ . The variational field equations arising from (2) are

$$\begin{aligned} D^B H^A &= 0, & D^B D^A \Phi_A &= [H^A, \Phi_B], \\ D^A K^B &= 0, & D^A D^B \Phi_B &= [K^B, \Phi_A]. \end{aligned} \quad (4)$$

By completing the square, the energy functional (2) can be rewritten as

$$\begin{aligned} 2\pi\mathcal{E} &= \int_{M_3} \langle (H^A \otimes I_E - I_E \otimes K^B) \wedge (D^A \Phi_A \otimes I_E - I_E \otimes D^B \Phi_B) \rangle \\ &\quad - \int_{M_3} \langle (D^A \Phi_A \otimes I_E) \wedge (H^A \otimes I_E) \rangle + (A \leftrightarrow B, \Phi_A \leftrightarrow \Phi_B). \end{aligned} \quad (5)$$

Let  $E_A$  and  $E_B$  denote the vector bundle  $E$  equipped with either the connection  $A$  or  $B$ , respectively. We recall that the curvature of the tensor product bundle  $E_A \otimes E_B^*$  is given by [9]

$$\Omega_{E_A \otimes E_B^*} = H^A \otimes I_E - I_E \otimes K^B.$$

By defining  $\Phi \equiv \Phi_A \otimes I_E - I_E \otimes \Phi_B$ , then  $D_{E_A \otimes E_B^*} \Phi = D^A \Phi_A \otimes I_E - I_E \otimes D^B \Phi_B$ . The first integral in (5) is now a topological invariant. The Bogomol'nyi equations arising from the energy functional (5) are

$$\begin{aligned} H^A \otimes I_E &= I_E \otimes K^B \\ D^A \Phi_A \otimes I_E &= I_E \otimes D^B \Phi_B \end{aligned} \quad (6)$$

The first equation in (6) is a zero curvature condition on the tensor product bundle  $E_A \otimes E_B^*$ . Reintroducing a coordinate system, we can rewrite the equations in (6) as

$$\begin{aligned} H_{ij} &= K_{ij} = F_{ij}(iI), \\ D_i^A \Phi_A &= D_i^B \Phi_B = E_i(iI). \end{aligned} \quad (7)$$

$F$  and  $E$  are a real-valued two-form and one-form on  $M_3$ , respectively, and  $I$  is the identity matrix. Solutions to the Bogomol'nyi equations (7) automatically satisfy the variational field equations (4). Solutions to the first equation in (7)

alone are well-known to differential geometers—they are the projectively flat connections [9]. For line bundles projective flatness is vacuous, but for bundles of rank greater than one projective flatness is a strong condition [9].

Unlike the theory of BPS magnetic monopoles, the first integral in the energy functional (5) vanishes with *either* Bogomol'nyi equation in (7) satisfied. This occurs because the first integral in (5) is not in the form of a perfect square (cf., Yang-Mills(-Higgs) theories). With one or the other Bogomol'nyi equation satisfied, the energy functional becomes the Bogomol'nyi energy. The Bogomol'nyi energy is found to be

$$2\pi\mathcal{E} = \int_{M_3} (D_{[k}^A \Phi_A)^a H_{ij]}^b \operatorname{tr}(T^a T^b) + \int_{M_3} (D_{[k}^B \Phi_B)^a K_{ij]}^b \operatorname{tr}(T^a T^b). \quad (8)$$

The extra flexibility in the topological field theory will lead us to electric monopoles.

### 3 Projectively-flat solitons

Presumably topological monopoles, if they exist, are analogous to the BPS magnetic monopole field configurations. Therefore we shall use the same symmetry breaking mechanism [10]. The solitonic core region is placed at the origin. Let  $G$  and  $G_o$  be compact and connected gauge groups, where the group  $G_o$  is assumed to be embedded in  $G$ . The gauge group of the core region  $G$  is spontaneously broken to  $G_o$  outside of the core region when the Higgs field is covariantly constant,  $D\Phi = 0$ . In regions far from the core where we assume that  $D^A \Phi_A = 0$ , it can be shown that

$$H^A = \Phi_A F_A, \quad (9)$$

where  $F_A \in \Lambda^2(M_3, E_{G_o})$ , a two-form on  $M_3$  taking values in the  $G_o$ -Lie algebra bundle, denoted by  $E_{G_o}$  here [10]. An equivalent expression to (9) can be written when  $D^B \Phi_B = 0$ . We shall assume that  $\langle \Phi_A^2 \rangle = \langle \Phi_B^2 \rangle = a^2$  when  $r \gg 1$  and where spontaneous symmetry breaking has occurred. When  $G = U(n)$  and  $G_o = U(1)$ ,  $F_A$  becomes a pure imaginary two-form on  $M_3$ . The Bogomol'nyi

solitons defined by (7) have an energy (8) topologically fixed by

$$\begin{aligned} 2\pi\mathcal{E} &= \int_{M_3} d(\text{tr}(\Phi_A H^A)) + \int_{M_3} d(\text{tr}(\Phi_B K^B)) \\ &= \int_{S^2} \text{tr}(\Phi_A H^A) + \int_{S^2} \text{tr}(\Phi_B K^B) \end{aligned} \quad (10)$$

where  $S^2$  is a large sphere surrounding the monopole core.

Substituting equation (9) into equation (10) and using the asymptotic normalization condition  $\langle \Phi^2 \rangle = a^2$  for both Higgs fields, the energy is fixed by  $a^2(\int F_A + \int F_B)/2\pi$ . Let us conventionally interpret  $a \int F_A/2\pi$  as the magnetic charge ( $g$ ), and  $a \int F_B/2\pi$  as the electric charge ( $q$ ). We can view  $F_A$  and  $F_B$  as curvatures on the line bundles  $L_A \rightarrow S^2$  and  $L_B \rightarrow S^2$  determined by  $\Phi_A$  and  $\Phi_B$ , because from equation (9)  $F_A$  and  $F_B$  are the projections of  $H^A$  and  $K^B$  on  $L_A$  and  $L_B$ . The magnetic and electric charges are proportional to topological invariants—the Chern numbers associated to complex line bundles with curvatures  $F_A$  and  $F_B$ , respectively. As a result both the solitonic electric and magnetic charges are quantized at the classical level. The Bogomol’nyi energy is given by

$$\mathcal{E} = a^2(c_1(L_A) + c_1(L_B)) = a(g + q). \quad (11)$$

The classical stability of the solitonic particle is also argued from the topological invariants. Stability is assured by (11) if either the electric or magnetic charge is non-vanishing.

Let us now consider non-singular, particle-like  $U(n)$  solutions to *both* Bogomol’nyi equations in (7) that are spontaneously broken in the far-field. From the projective flatness of the curvatures in the Bogomol’nyi equations (7),  $H^A = K^B = F(iI)$ , and from equation (9) we conclude that  $F = \varphi_A F_A = \varphi_B F_B$ , where  $\Phi_A = \varphi_A(iI)$ ,  $\Phi_B = \varphi_B(iI)$  and  $\varphi_A, \varphi_B$  are real-valued functions on  $M_3$ . The normalization of the Higgs fields implies that  $\varphi_A^2 = \varphi_B^2 = a^2$ . From this we find that

$$\int_{S^2} F_A = \pm \int_{S^2} F_B. \quad (12)$$

Therefore non-singular, stable, particle-like solutions to both Bogomol’nyi equations (7) are dyons.

To obtain electric monopoles there would appear to be two possibilities, both resulting from a weakening of one or the other Bogomol'nyi equation (7). We do not favour relaxing the projective flatness of the solitons, however, because we then lose mathematical control over the nice topological properties of the configuration space [9]. (The configuration space is Hausdorff, presumably.) Instead, we shall maintain projective flatness and let go of  $D^A\Phi_A = D^B\Phi_B = E(iI)$ , at least asymptotically. Furthermore, since little empirical evidence exists to suggest the independent existence of two gauge potentials, it is desirable to restrict to a smaller set of topological solitons defined by  $A = B$ . We shall call these solutions 'diagonal projectively-flat solitons'. For diagonal projectively-flat solitons the second-order variational field equations become the Bianchi identities, and are therefore automatically satisfied.

## 4 Diagonal projectively-flat electric monopoles

In this section we demonstrate the existence of diagonal projectively-flat  $U(2)$  electric monopoles ( $A = B, \Phi_A, \Phi_B$ ) on  $\mathbf{R}^3$  situated at the origin. Following the example of the BPS magnetic monopole we define the outside of the monopole to be where the gauge field is broken with a covariantly constant Higgs field [10]. We assume the following properties for the electric monopole:

1.  $A = B$  are projectively flat on all of  $\mathbf{R}^3$  and take values in the Lie algebra of  $U(2)$ ;
2.  $A = B$  are asymptotically flat on  $\mathbf{R}^3$ ;
3.  $\Phi_A, \Phi_B$  are any sufficiently differentiable functions on  $\mathbf{R}^3$  taking values in the Lie algebras of  $SU(n)$  and  $U(n)$ , respectively;
4.  $D^B\Phi_B = 0$  and  $D^A\Phi_A \neq 0$ , asymptotically. When  $D^B\Phi_B = 0$ , we assume that  $\Phi_B = aI_E$  for a non-zero constant,  $a$ .

5. The electric charge of the monopole is non-zero, and the magnetic charge vanishes.

Condition 4. is equivalent to stating that the gauge symmetry for  $K^B$  is broken to  $U(1)$  asymptotically, and that the gauge group for  $H^A$  is not permitted to break far from the origin. For condition 5., assume that a two-sphere of radius  $r$ ,  $S_r^2$ , lies completely outside the monopole that is centered at the origin. The Bogomol'nyi energy (10) is then given by

$$\mathcal{E} = -\frac{1}{2\pi} \int_{S_r^2} F_B \operatorname{tr}(\Phi_A \Phi_B) - \frac{a^2}{2\pi} \int_{S_r^2} F_B. \quad (13)$$

The first integral in (13) is the magnetic charge. The magnetic charge vanishes since  $\Phi_A$  is traceless and  $\Phi_B = aI_E$ , conditions 3. and 4., respectively. The second term is the topological electric charge; the electric charge must be non-zero. We shall show that a solution satisfying conditions 1. through 5. exists.

In general, a  $U(2)$  diagonal topological soliton ( $A = B, \Phi_A, \Phi_B$ ) must be of the form

$$A_j = B_j = \begin{pmatrix} ia_j & -c_j^* \\ c_j & ib_j \end{pmatrix}, \quad (14)$$

$$\Phi_A = \begin{pmatrix} i\alpha_A & -\gamma_A^* \\ \gamma_A & i\beta_A \end{pmatrix}, \Phi_B = \begin{pmatrix} i\alpha_B & -\gamma_B^* \\ \gamma_B & i\beta_B \end{pmatrix}.$$

$a_j, b_j, \alpha_A, \beta_A, \alpha_B, \beta_B$  are all real-valued functions on  $\mathbf{R}^3$ . The first Bogomol'nyi equation in (7) states that the vector potential is projectively flat,  $H^A = K^B = F(iI)$ . A straightforward calculation of  $H^A$  informs us that

$$\begin{aligned} F_{ij} &= \partial_i a_j - \partial_j a_i + i(c_j^* c_i - c_i^* c_j), \\ &= \partial_i b_j - \partial_j b_i - i(c_j^* c_i - c_i^* c_j). \end{aligned} \quad (15)$$

and that

$$\partial_i c_j - \partial_j c_i - i[(c_i a_j - c_j a_i) - (c_i b_j - c_j b_i)] = 0. \quad (16)$$

Equation (15) in coordinate free notation becomes

$$F = d\mathbf{a} + i\mathbf{c}^* \wedge \mathbf{c} = d\mathbf{b} - i\mathbf{c}^* \wedge \mathbf{c}, \quad (17)$$

and implies that  $\mathbf{c}^* \wedge \mathbf{c} = 2id(\mathbf{b} - \mathbf{a})$ , that is,  $\mathbf{c}^* \wedge \mathbf{c}$  is exact. Similarly, equation (16) can be rewritten as

$$d\mathbf{c} + i(\mathbf{a} - \mathbf{b}) \wedge \mathbf{c} = \mathbf{0}, \quad (18)$$

where  $\mathbf{a} = a_i dx^i$ ,  $\mathbf{b} = b_i dx^i$ , and  $\mathbf{c} = c_i dx^i$ . We must now write down a solution to equations (17) and (18) that can be shown latter to have non-vanishing electric charge.

We introduce the complex coordinate  $\zeta = \mathbf{P}_r(r, \theta, \varphi)$  that comes from the stereographic projection,  $\mathbf{P}_r$ , of the spherical polar coordinate  $(r, \theta, \varphi)$  on the two-sphere minus the north pole,  $S_r^2 - \{N\}$ , to the complex plane minus infinity,  $\mathbf{CP}^1 \setminus \{\infty\}$ . The projected 1-forms  $\mathbf{a}$  and  $\mathbf{b}$  on  $\mathbf{CP}^1 \setminus \{\infty\}$  will also be denoted by  $\mathbf{a}$  and  $\mathbf{b}$ . Assume that  $\mathbf{a}$  and  $\mathbf{b}$  on  $\mathbf{CP}^1 \setminus \{\infty\}$  differ by the non-exact real-valued 1-form,  $\chi$ ; that is,  $\mathbf{b} = \mathbf{a} + \chi$ . Now define

$$\mathbf{c}(\zeta) = i\sqrt{2} \left[ \exp i \int_{\mathbf{P}_\gamma} (\mathbf{b} - \mathbf{a}) \right] d\zeta, \quad (19)$$

where the contour integration is along the stereographic projection of the great circle,  $\gamma$ , from the south pole of  $S_r^2$  to the spherical polar coordinate given by  $(r, \theta, \varphi) = \mathbf{P}_r^{-1}(\zeta)$ . Assume that both the 1-forms  $\mathbf{a}$  and  $\mathbf{b}$  vanish at the south pole for all values of  $r$ . It is easy to verify that (19) satisfies equation (18), and that  $\mathbf{c}^* \wedge \mathbf{c} = 2i d\chi \equiv -2d\bar{\zeta} \wedge d\zeta$ .

Turning to the Higgs fields, recall that we require that  $A$  be  $su(2)$ -valued everywhere, and that the gauge group for  $B$  is broken to  $U(1)$  asymptotically ( $D^B \Phi_B \rightarrow 0$  as  $r \rightarrow \infty$ ). Since the Higgs field  $\Phi_A$  takes values in the Lie algebra of  $SU(2)$ , then  $\|\gamma_A\|^2 = 1 + \alpha_A \beta_A$ . To demonstrate the existence of an electric monopole, take  $\alpha_A = \beta_A = 0$  so that  $\|\gamma_A\|^2 = 1$ . For  $\Phi_B$ , the only condition imposed on the Higgs fields is that  $\Phi_B = aI_E$ , asymptotically. Substitute (14) into the asymptotic equation  $D^B \Phi_B = 0$  to find that on the diagonal

$$d(\alpha_B + \beta_B) = 0, \quad d\alpha_B = i(\mathbf{c}\gamma_B^* - \mathbf{c}^*\gamma_B), \quad (20)$$

and off the diagonal,

$$d\gamma_B = i((\beta_B - \alpha_B)\mathbf{c} + (\mathbf{a} - \mathbf{b})\gamma_B), \quad \text{complex conj.} \quad (21)$$

For  $|\zeta| \gg 1$ , let

$$\begin{aligned} \alpha_B(\zeta, \bar{\zeta}) &= a + \frac{1}{(1 + |\zeta|^2)^2} + O((\bar{\zeta}\zeta)^{-3}), \\ \beta_B(\zeta, \bar{\zeta}) &= a + \frac{1}{(1 + |\zeta|^2)^2} + O((\bar{\zeta}\zeta)^{-3}). \end{aligned}$$

$\alpha_B = \beta_B = a$  asymptotically is consistent with the requirement that  $\Phi_B = aI_E$ , and satisfies the asymptotic equations (20) and (21). The remaining asymptotic equations for  $\gamma_B$  become

$$\begin{aligned} c_i \gamma_B^* - c_i^* \gamma_B &= 0, \\ \partial_i \gamma_B + i \gamma_B (b_i - a_i) &= 0. \end{aligned}$$

$\gamma_B = 0$  is a solution to these equations and is also consistent with the requirement that  $\Phi_B = aI_E$ .

Now we compute the electric charge. The broken gauge far-fields are given by

$$\begin{aligned} F_A &\equiv \langle H^A \Phi_A \rangle = -iF \text{tr}(\Phi_A) = 0, \\ F_B &\equiv \langle K^B \Phi_B \rangle = -iF \text{tr}(\Phi_B) = F(\alpha_B + \beta_B)/2, \end{aligned} \quad (22)$$

Recall that the magnetic charge of the solution, given by  $\lim_{r \rightarrow \infty} a \int F_A / 2\pi$ , vanishes because  $\Phi_A$  is traceless. The solitonic electric charge is given by

$$2\pi q = a \lim_{r \rightarrow \infty} \int_{S_r^2} F_B = -2ia \int_{\mathbf{CP}^1} \frac{d\bar{\zeta} \wedge d\zeta}{(1 + |\zeta|^2)^2} = -4\pi a,$$

where  $\mathbf{a}$  is closed, but  $\mathbf{b}$  is not closed. The sphere  $S_r^2$  is centered round the monopole at the origin and is assumed to lie completely in regions where the gauge field has been broken. Notice that the electric charge within  $S_r^2$  depends only on the gauge potential, and is time-independent, gauge-invariant, and unchanged under any continuous deformation of the enclosing surface. This proves the existence of an electric monopole within this theory.

To interpret these solitons, we note that  $U(2)$  is double covered by  $SU(2) \times U(1)$ . Also, the classical mass and particle spectrum of the projectively-flat electric monopole compare favourably with that of the intermediate vector bosons. The mass of the electric monopole is  $M = aq$ , the same as the mass of the  $W^\pm$ . Moreover, there are no quantum corrections to the classical mass, because of the general relationship between supersymmetry and the Bogomol'nyi structure [1][2]. The  $Z_0$ , presumably, corresponds to the case where  $H^A = K^B = FI_E$ , but  $D^A\Phi_A \neq 0$  and  $D^B\Phi_B \neq 0$ , that is, there is no symmetry breaking. The gauge far-fields in that case are non-abelian and pass unnoticed through the detectors. Although uncharged the soliton's energy is topologically fixed by

$$\begin{aligned} 2\pi\mathcal{E} &= \int_{S^2} \text{tr}(\Phi_A H^A) + \int_{S^2} \text{tr}(\Phi_B K^B) \\ &= -\int_{S^2} F < \Phi_A > - \int_{S^2} F < \Phi_B > \end{aligned}$$

So it, too, is stable under perturbations. We propose therefore that the solitons in the tensor product topological field theory defined in section two form a provisional model for the  $W^\pm$  and  $Z_0$  intermediate vector bosons. Many years ago it was promoted that intermediate vector bosons should appear as Bogomol'nyi solitons dual in some sense to the BPS magnetic monopole [11].

## 5 Conclusion.

The tensor product energy functional (2) has a Bogomol'nyi structure and solitonic particle solutions. The projective-flatness observed in the first Bogomol'nyi equation (7) is well-known to be related to algebraic stability in complex vector bundles, and used in the construction of well-behaved moduli spaces [9]. Moduli spaces in this context are often seen to be the covariant phase spaces and configuration spaces of the physical theory, so it is not surprising to uncover a requirement for projective-flatness, although such a requirement is not necessary. As the Bogomol'nyi equations do not arise from a perfect square, there is more flexibility in achieving the Bogomol'nyi energy. An interesting

class of soliton has been studied herewithin by restricting to those solutions of the Bogomol'nyi equations where the gauge potentials are equal,  $A = B$ . When both Bogomol'nyi equations are satisfied stable, particle-like solitons are dyonic. By relaxing the Bogomol'nyi structure stable, particle-like solutions can be found that carry only an electric charge. Similarities exist between the heavy intermediate vector bosons in the standard model and the solitons in this theory.

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